THERMAL PARAMETERS AS A FUNCTION OF THICKNESS FOR COMBINED RADIATION AND CONDUCTION HEAT TRANSFER IN LOW-DENSITY INSULATION

Brian G. Rennex

ABSTRACT

Approximate expressions are obtained for the apparent thermal conductivity and thermal resistance, in the case of combined conductive and radiative heat transfer through low-density insulation. These expressions are obtained in the regimes of intermediate and large optical depth, and they depend on only two parameters to be determined by experiment, namely the "conduction" thermal conductivity, k_c, and the extinction coefficient, β. An expression is obtained for the apparent thermal resistance of an insulation consisting of layers with distinct densities. The applicability of these expressions to the interpretation of experimental results and to the improvement of ASTM Test Methods is discussed.

Key words: Conduction heat transfer, experimental aspects, linear superposition, multi-layer heat transfer, radiation heat transfer, thickness effect,

INTRODUCTION

There is considerable interest in the effect of thickness on the "apparent" thermal conductivity of low-density insulation materials. References 1-27 represent work related to this topic pursued over the last twenty years. The works considered by the author to be most pertinent to this publication begin with that by Larkin and Churchill (1959) [2]. Using the two-flux model for radiant transfer, which takes into account scattering, absorption and re-emission of radiation in a grey medium they arrived at a set of differential equations, the solution of which enables one to calculate the radiative heat flux. For pure scattering, these equations have the exact solution for a semi-infinite parallel slab,

$$q_r = \frac{\sigma(T_2^4 - T_1^4)}{2/\varepsilon - 1 + NL},$$

where q_r is the radiative heat flux, ε is the emittance of the slab faces, N is the back-scattering cross section and L is the slab thickness. When absorption is included, there is no closed-form solution of the differential equations, and one must resort to numerical methods to calculate the values of the heat flux. Larkin and Churchill also used curve-fitting techniques on transmission data to evaluate the absorption and back-scattering cross sections at temperatures above 200°F. They found the back-scattering cross section to be 80 to 100 times greater than the absorption cross section at 200°F.

Heaslet and Warming [11] performed a numerical calculation of radiative heat transfer and provided tables of the dimensionless heat flux, Q, as a function of optical depth, τ_0. Their results are considered to be accurate enough to be called exact [12].

B. R. Rennex is a Physicist in the Thermal Insulation Performance Section of the Center for Building Technology at the National Bureau of Standards, Washington, D.C. 20234. This work was written while Dr. Rennex was working for the National Association of Home Builders Research Foundation.

Viskanta (1965) [3] considered the case of combined radiation and conduction in a grey medium. He obtained two simultaneous, nonlinear, integro-differential equations describing the heat transfer. Numerical methods must be used to obtain solutions, and these solutions depend on a number of independent parameters. Viskanta mentioned the excessive number of iterations necessary to obtain accurate numerical solutions and emphasized the need for "simple but physically realistic models" to describe heat transfer in low density materials.

Pelanne (1968) [4] made an experimental study of the various components of both radiative and conductive heat transfer. The radiative components consisted of radiation transmission, radiation conduction and a term due to the interaction between air and fiber conduction.

In this author's (Rennex) opinion, the term referred to by Pelanne as the radiation transmission thermal conductivity,*

$$k_{rt} = \frac{\sigma(T_h^4 - T_c^4)\ L/(T_h - T_c)}{R + (\frac{1}{\varepsilon} - 1)},$$

includes the radiation conduction term, k_{rc}, as well. This would imply that k_{rc} (Pelanne) = 0 and k_{rt} (Pelanne) = $k_{rc} + K_{rt}$. This, in turn, implies that the "solid-conduction" term, k_s, must be larger than was speculated in Pelanne's paper (by eq. 3 or 5).

The above interpretation negates some conclusions of Pelanne's paper, but the conclusion that the sum, k_i (interaction) + k_c (convection), accounts for 4% to 11% of the total k at a density of about 1 lb/ft^3 seems to be still valid. Pelanne's Fig. 5 implies that the interaction term is larger in the case of the gold plates (k_{ig}) than in the case of the black plates (k_{ib}). This is expected according to Fig. 6 in Viskanta's paper [3], which shows that a smaller plate emissivity results in a larger non-linearity of the temperature distribution. Since the "k_i" term is thought to result from this non-linearity, it would be expected to be larger in the "gold" case.

Assuming that the order of magnitude of k_i is 5% to 10% of the total k (Pelanne's result), then there must be s significant absorption, within the sample, of the thermal radiation--because, the radiation scattering is not expected to distort the temperature distribution from linearity (since it cannot transform radiant energy into thermal energy). Thus, the above assumption is tantamount to there being a significant thermal-radiation absorption within the sample.

Then, the question is whether the glass or the gas is causing the absorption. Fig. 3 and 4 of Pelanne's paper [4] offer an interesting possibility for the answer to this question. The difference of the total thermal conductivity between the "gold" and "black" case is larger for vacuum pressures than for atmospheric pressures. A possible explanation for this phenomena assumes that the gas (air) is causing a significant part of the thermal radiation scattering. In order to see how this explanation goes, consider the following equation (which is explained in the Radiant Heat Flux Section).

$$q_{total} = q_c + \frac{\text{constant}}{3/4\beta L + 2/\varepsilon - 1}$$

The extinction coefficient, β, is the sum of the scattering and absorption coefficients. If the thermal radiation absorption occurs in the air, then β is smaller in the vacuum case than in the atmospheric pressure case. The "$2/\varepsilon - 1$" term is much larger in the "gold" case than in the "black" case, which means that the absorption coefficient is less of a determining factor in the denominator of the expression for q_{total}. By working an example, one can see that the difference between q_{total} (black) and q_{total} (gold) is expected to be larger in the case of a vacuum than in the case of atmospheric pressure. This agrees with Pelanne's results. One would hope that the interesting work begun by Pelanne will be expanded upon by others in an effort to better understand the above-described phenomena.

Dayan and Tien (1975) [20] solved the problem of radiative transfer in an absorbing and linearly-anisotropic scattering grey medium, both numerically and in closed form. They also approximated the temperature distribution within the medium with a polynomial, for the case of combined radiation and conduction.

* Note that the term in the denominator should be $R + 2(\frac{1}{\varepsilon} - 2)$, for R to be optical depth.

Lao and Skochdopole (1976) [7] used a numerical method to solve for the temperature and its first three derivatives as a function of distance across the slab, starting with the two-flux model. They then made an approximation that the third derivative of temperature with respect to distance is zero, and they arrived at an analytical solution of the conductivity as a function of thickness. It is not apparent to the author that their assumption is always an improvement over the usual assumption of a linear dependence of temperature on distance. A small problem with their "large-separation" approximation is that it does not have a necessary factor of 3/4. (This is due to a subtle feature of differential equations described in reference 18.)

Pelanne (1978) [8] discussed results of an experiment measuring the conductivity as a direct function of density and as an indirect function of thickness (using paper septums). This experiment demonstrated the existence of a "thickness" effect (on the measured thermal conductivity due to thermal radiation), as did another experiment by Hollingsworth [9]. Pelanne (1978) [19] also measured optical properties of fiberglass material and noted a correlation between light transmission through fiberglass and apparent conductivity of fiberglass. Striepens (1978([13] made transmission measurements over a wide range of temperatures and pressures and noted a good correlation between experimental results and model predictions based on the optical parameters. The model assumed the radiation and conduction heat transfer modes to be independent. Bhattacharyya (1978) [10] presented data confirming the postulate that the back-scattering coefficient, N, is linearly dependent on the fiberglass bulk density.

Sparrow and Cess (1978) discussed the integral equation that describes radiation transfer in a grey medium under equilibrium and noted that the absorption and scattering coefficients appear only as their sum, the extinction coefficient, β, in the integral equation describing the temperature field in the medium. (Page 215 of reference 12.)

RADIATIVE HEAT FLUX

Here, we derive the expression for the heat flux in the case that only the radiation mode is operative. Using the two-flux model [12], the equilibrium heat flux is

$$q_r = \frac{\sigma(T_2^{\,4} - T_1^{\,4})}{1/Q + 2/\varepsilon - 2} \tag{1}$$

assuming that the surface emittances of both the hot and cold plates are given by $\varepsilon = \varepsilon(T_1) = \varepsilon(T_2)$.*

The dimensionless heat flux is defined by the equation, $Q = {}^{q_r}/{}_{B_1 - B_2}$, where B_i is a surface radiosity. Q is a function only of the optical depth, $\tau o = \int_0^L \beta(T,\rho)dx$.** The exponential kernel approximation results in the following expression for Q,

$$Q[EK] = \frac{1}{1 + 3/4\tau o} \tag{2}$$

Heaslet and Warming [11] used numerical techniques to obtain tabulated values of Q as a function of τo. These values are accurate enough to be considered "exact" [12]; i.e., the numerical solutions to the differential equations are extremely accurate. Table 1 shows that the exponential kernel approximation (eq. 2) for Q agrees with the "exact" expression for Q to within 4% for intermediate values of τo and to within 0.5% for $\tau o > 16.5$.

This paper proposes the following approximate expression for Q,

$$Q[TH] = \frac{1}{1 + 3/4\tau o + 0.0657\tanh(2\tau o)}. \tag{3}$$

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* This is for the case of radiative heat transfer thorugh a grey medium between two infinite parallel surfaces at temperatures T_1 and T_2.

** The extinction coefficient β is the sum of the absorption and scattering coefficients and is a function of the density and the temperature distribution.

Table 1 shows that Q[TH] agrees with Q exact to within $\pm$ 0.5% over the entire range of τo, and Q[TH] $\rightarrow$ Q exact in the limits of $\tau o \rightarrow 0$ and $\tau o \rightarrow \infty$.

Heaslet and Warming [11] proposed the approximate expression for Q,

$$Q[HW] = \frac{1}{3/4\tau o + 1.0657}. \tag{4}$$

Table 1 shows that Q[HW] agrees with Q exact to within 0.5% for $\tau o > 1$.

Thus, there are three approximations for the dimensionless heat flux -- the exponential kernel approximation using (2), the Heaslet and Warming approximation using (4), and the TANH approximation using (3). Using (1), these expressions give the following approximations for the heat flux.

$$q_r[EK] = \frac{\sigma(T_2^{\ 4} - T_1^{\ 4})}{2/\varepsilon - 1 + 3/4\tau o} \tag{5}$$

$$q_r[HW] = \frac{\sigma(T_2^{\ 4} - T_1^{\ 4})}{2/\varepsilon - 1 + 3/4\tau o + 0.0657} \tag{6}$$

$$q\ [TH] = \frac{\sigma(T_2^{\ 4} - T_1^{\ 4})}{2/\varepsilon - 1 + 3/4\tau o + 0.0657\tanh(2\tau o)} \tag{7}$$

If one assumes that an agreement within 0.5% with the exact heat flux is the criterion for experimental applicability, then the range over which the various approximations apply are the following:

$q_r[EK]$ (exponential kernel approximation) applies for $\tau o > 16.5$,
$q_r[HM]$ (Heaslet and Warming approximation) appliex for $\tau o > 1$, and
$q_r[TH]$ (TANH approximation) applies for all τo.

The optical depth τo is, strictly speaking, a function of wavelength. In this case, it is approximated by an average value over the thermal radiation wavelength range. This range is determined by the blackbody radiation spectral intensity curve (~3 - 30μ; the radiated power per wavelength is less than 10% of the maximum power per wavelength outside this range for T = 75°C) and by the extinction coefficient special curve in fiberglass. Unfortunately, the author is not aware of published data on the latter curve.

Note that the absorption and scattering coefficients do not appear separately in the equations for the radiative heat flux; but, rather they appear as their sum, the extinction coefficient β.

$$\tau o = \int_0^L \beta dx$$

This is a feature of the model employed by Heaslet and Warming [11]. Sparrow discusses this feature on pages 215 and 238 of reference 12. He also noted that the extinction coefficient β is not assumed to be a constant, but rather, can be calculated as a function of the temperature, T.

Thus, if the extinction coefficient is known as a function of temperature and if the temperature distribution T(x) is known, one could more accurately calculate

$$\tau o = \int_0^L \beta(T)dx$$

Larkin and Churchill [2] measured $\beta(T)$ at temperatures greater than those appropriate to fiberglass house insulation testing, so $\beta(T)$ is not well known in this application. Dayan and Tien [20] have obtained a polynomial expression for T(x). However, the above-mentioned refinement in τo is expected to be small compared to experimental errors.

The results in (4) can be extended to take into account the work of Dayan and Tien [20] in which they calculated the radiative heat flux taking into account anisotropic scattering and albedo. They described the linear anisotropic scattering by p (cos θ_o) = 1 + x cos θ_o ($-1 < x < 1$) where p is the scattering phase function, θ_o is the angle between the incident and scattered ray and x goes to +1 for strong forward scattering and -1 for strong back-scattering.

By matching thin and thick optical depth solutions, they arrived at an approximate expression for the dimensionless heat flux Q,

$$Q = \frac{1}{1 + (3/4 - \frac{w_o x}{4})\tau o} \tag{8}$$

where $w_o = N/\beta$, is the single scattering albedo and represents the fraction of incident radiation that is scattered. N is the scattering coefficient.

This expression can be extended by comparison with (3) to

$$Q = \frac{1}{1 + (3/4 - \frac{w_o x}{4})\tau o + m\tanh(n\tau o)} \tag{9}$$

where m and n are constants to be determined by fitting to an "exact" theoretical curve. In this case, the heat flux q_r is given by

$$q_r = \frac{\sigma(T_2^4 - T_1^4)}{2/\varepsilon - 1 + (3/4 - \frac{w_o x}{4})\tau o + m\tanh(n\tau o)} \tag{10}$$

Note that in the case of fiberglass insulation, it is expected that $w_o \approx 1$. The author is not aware of published results on the anisotropy parameter, x, for fiberglass.

COMBINED RADIATION AND CONDUCTION HEAT FLUX

The conductive heat flux is described in its most simple form by

$$q_c = k_c \frac{\Delta T}{L} \tag{11}$$

where k_c is the thermal conductivity, $\Delta T = T_2 - T_1$ is the temperature difference between the hot and cold plates and L is the slab thickness. More complicated expressions take into account both the porosity and the series-and-parallel configuration of the modes of heat transfer through the gas and the solid fibers [10 and 13-17]. In the interest of minimizing the number of experimental parameters, (11) is used to describe the thermal conductivity heat transfer. The parameter k_c is a function of the gas conductivity, the fiberglass conductivity and the density of the fiberglass.

The most simple way to describe the combined radiative and conductive modes of heat transfer is to assume that the modes are independent.* This ignores the fact that the radiative mode affects the conductive mode by changing the temperature distribution, and vice-versa. Thus, the expression for q_c (11), is simply added to the expression for q_r, (5), (6) or (7), to obtain the combined heat flux, q, for the three approximations. For example, using (6) for the Heaslet-Warming approximation,

$$q[HW] = k_c \frac{\Delta T}{L} + \frac{\sigma(T_2^4 - T_1^4)}{2/\varepsilon - 0.9343 + 3/4\tau o} \tag{12}$$

Using the equation

$$k = \frac{L}{\Delta T} \tag{13}$$

* Note that convection (within the air cells) is not considered here, as it is thought to be negligible at densities above 0.1 Lb/Ft^3 [27]. Convection due to large scale gas movement within a sample probably is not negligible if the sample is thick enough, the density is small enough and the ΔT is large enough; nevertheless, this latter convection is too complicated for the scope of this paper.

and

$$R = L/k$$

one can define thermal conductivities and resistances corresponding to q_c, q_r and q.

Since the heat transfer modes are assumed to be independent, the combined thermal resistance is computed as follows:

$$\frac{1}{R} = \frac{1}{R_c} + \frac{1}{R_r} \tag{14}$$

When the approximate expression for q_r, (5), (6) or (7), is used, one can calculate R. In the case of low-density mineral wool insulation at room temperature, $\tau o > 1$, and the Heaslet-Warming approximation applies.

$$R[HW] = \frac{L}{k_c} \quad \frac{2/\varepsilon - 0.9343 + 3/4\beta L}{2/\varepsilon - 0.9343 + 3/4\beta L + \frac{4\sigma\bar{T}^3 L}{k_c}} \tag{15}$$

Here $\tau o = \beta L$ and $4\bar{T}^3 = (T_2 + T_1)(T_2^2 + T_1^2)$.

In the limit of small L ($L \leq 0.1$ inch for fiberglass), the L^2 term in (15) is ignored and $R_t(L)$ has the functional form,

$$\lim_{L \to 0} R(L) \to \frac{1}{1 + 1/L} \tag{16}$$

That is, R climbs steeply and then bends over to a smaller slope (See Fig. 1a). In the limit of large L, the L^2 term in (15) becomes dominant and the combined thermal resistance has the functional form,

$$\lim_{L \to \infty} R(L) \to A + BL \tag{17}$$

The exact expression for the R(L) limit is

$$\lim_{L \to \infty} R(L) \to \underbrace{\frac{(2/\varepsilon - 1 + 0.0657)/4\sigma\bar{T}^3}{k_c/k_s + 1}}_{A} + \underbrace{\frac{1}{k_c(1 + k_s/k_c)}}_{B} L \tag{18}$$

where the effective scattering conductivity is given by $k_s = \frac{4\sigma\bar{T}^3}{3/4\beta}$.

Note that a is less for greater radiation transmission or scattering and less for greater thermal conductivity.* In order to project R(L) for larger L, even in the conduction regime (L large), one must measure R(L) for two values of L in order to determine the two parameters--either a and b, k_s and k_c or k_c and β. This is an important point in reference to the ASTM Test Method C518 to determine thermal parameters.

Using (12), the combined thermal conductivity is

$$k(L) = k_c + k_s \left[\frac{1}{1 + \frac{k_s}{4\sigma\bar{T}^3}(2/\varepsilon - 0.9343)}\right] \tag{19a}$$

Figure 2a shows this curve. Figure 2b shows the curve of the inverse of k(L) (this is the resistivity, $r = \frac{R}{L}$). A common value for β in low-density mineral wool insulation is 8 $(\text{In})^{-1}$.

* For $\beta = 5.5\ \text{In}^{-1}$, $A = .85\ \frac{\text{Ft}^2\text{-Hr-F}}{\text{Btu}}$; for $\beta = 11\ \text{In}^{-1}$, $A = .51\ \frac{\text{Ft}^2\text{-Hr-F}}{\text{Btu}}$.

An important number, in relation to the ASTM C518 and C177 Test Methods to determine thermal parameters, is the so-called representative thickness at which the thermal conductivity is within 2% of the infinite-L conductivity. That is, $k(L) > 98\%$ of $k(\infty)$. Table 2 gives this thickness for various densities using various values for β and the Heaslet-Warming approximation of (19a):

$$k(L) = k_c + \frac{ks}{1 + (G/L)ks}$$

$$\text{where } G = \frac{(2/\varepsilon - 0.943)}{4\sigma T^3} = 1.23 \ \frac{\text{Hr-Ft}^2\text{-°F}}{\text{Btu}} \tag{19b}$$

for $T_1 = 55°F$ and $T_2 = 95°F$.

This equation could be used either for interpolation or extrapolation to determine the representative thickness in the ASTM C518-76$^\varepsilon$ and C177-76$^\varepsilon$ Test Methods. The details of this suggestion are discussed in [25].

In order to arrive at the form of the expression for the combined thermal conductivity k as a function of density D, it is assumed that the thermal conductivity k_c and the extinction coefficient β are linearly dependent on the density,

$$k_c = a + bD \tag{20}$$

(See page 302 of [13])

$$\text{and } \beta = \beta' D \text{ (from [10])} \tag{21}$$

(The author expects that future research will indicate a more complicated dependence of β on D.)

$$k(D) = a + bD + \frac{4\sigma \bar{T}^3}{3/4\beta' D + \frac{1}{L}(2/\varepsilon - 0.9343)} \tag{22}$$

k can be written as:

$$k(D) = a + bD + \frac{c'}{D + f/L}, \tag{23}$$

where $c' = 4\sigma\bar{T}^3/(3/4\beta')$ and

$$f = \frac{2/\varepsilon - 0.9343}{3/4\beta'}$$

This curve is plotted in Fig. 3. For small D, the curve does not go to infinity, but rather it bends over and approaches the value of $a + c'L/f$. This is in the radiation transmission regime.

With regard to the ASTM C653-70 Test Method to extrapolate using a "k vs D' curve, the density and the thickness are related by the equation $D = 12W/AL$, where W is the sample weight and A is the sample area in the heat-flow meter area.* This assumes that none of the bulk within the heat-flow-meter area is squeezed out of that area when L is decreased; that is, it assumes that W/A stays constant. Therefore, in this application one can eliminate L from (23) to obtain:

$$k(D) = a + bD + \frac{c}{D} \tag{24}$$

$$\text{where } c = \frac{c'}{1 + fA/12W}.$$

This is, in fact, the form of the equation used in ASTM C653-70. Thus, for C653 the current form of the equation takes into account the "effect of thickness" on the apparent thermal conductivity.

* L units are In.

happen. Then, the thermal conductivity in the case of the sample of width, L, would not, in general, be expected to be the same as in the case of the thicker sample with septums at intervals of L. Thus, the good agreement between these two cases in Pelanne's paper provides experimental support of the assumption of linear superposition. Certainly, further experiments should be performed to answer the questions associated with this subtle and complicated phenomena.

COMBINED RADIATION AND CONDUCTION IN A MULTI-LAYER SLAB

Consider the problem of combined radiative and conductive heat transfer in a slab consisting of layers of material with distinct densities, D_i, and thermal conductivities, k_{ci}, (See Fig. 4). Assuming that the radiation mode is independent of the conduction mode, then the thermal resistance associated with conduction, R_c, is added in parallel with the thermal resistance associated with radiation, R_r, to obtain the total thermal resistance, R_t.

$$R_t = \frac{R_r R_c}{R_r + R_c} \tag{25}$$

The total resistance due to conduction, R_c, is obtained by adding (in series) the thermal resistance due to conduction of each layer.

$$R_c = \sum_i R_{ci} \tag{26}$$

The total resistance due to radiation, R_r, is obtained by adding each layer's contribution to the optical depth,

$$\tau_o = \sum_i \tau_{oi} = \sum_i \beta L_i \tag{27}$$

This total optical depth is then substituted into (6) for q_r and an effective radiative thermal resistance can be calculated using $R_r = L/(q_r L/\Delta T)$.

$$R_r = \frac{2/\varepsilon - 0.9343 + 3/4\Sigma_i \beta_i L_i}{4\sigma \bar{T}^3} \tag{28}$$

If we define an average thermal conductivity, $\bar{k}_c = L/\Sigma_i L_i/k_{ci}$, (29)
and an average extinction coefficient, $\bar{\beta} = \Sigma_i \beta_i L_i/L$ (30)
then the same forms of (15) and (18) can be used to obtain the total R_t as in the case of the single layer slab.

For example, from (15)

$$R_t(L) = \frac{L[2/\varepsilon - 0.9343 + 3/4\bar{\beta}L]}{\bar{k}_c[2/\varepsilon - 0.9343 + 3/4\bar{\beta}L + \frac{4\sigma\bar{T}^3 L}{\bar{k}_c}]} \tag{31a}$$

and, from (18) for the limiting value as L becomes large,

$$R_t(L) \underset{L \to \infty}{\to} \frac{2/\varepsilon + 0.0657}{4\sigma\bar{T}^3(k_c/\bar{k}_s + 1)} + \frac{L}{\bar{k}_c(1 + \bar{k}_s/\bar{k}_c)} \tag{31b}$$

where $\bar{k}_s = 4\sigma\bar{T}^3/3/4\bar{\beta}$. Note that the average thermal conductivity, $\bar{k}_c$, and the average extinction coefficient, $\bar{\beta}$, are now functions of the layer thicknesses, L_i.

Or equivalently,

$$k_t(L) = \bar{k}_c + \frac{\bar{k}_s}{1 + \frac{G}{L}\bar{k}_s}, \tag{32}$$

Detailed suggestions for the revision of the ASTM C653 Test Method, based on (24), are given in [26].

DISCUSSION OF LINEAR SUPERPOSITION OF CONDUCTIVE AND RADIATIVE HEAT FLOWS

In the previous section, a linear superposition of the conduction and radiation heat transfer modes was assumed in order to arrive at expressions simple enough for use in experiments or quality control. Here is discussed the information currently in the literature pertaining to the question, "How good is this assumption?"

Sparrow and Cess (on pages 256 of [12]) present the integral equations (first derived by Viskanta [3] which serve as the solution of the problem when conduction and radiation are not independent. These are difficult integral equations requiring a numerical solution. Lick [21] used the exponential kernel approximation to solve the problem when conduction and absorption (but not scattering) are combined dependently. Sparrow and Cess noted (on page 267 of reference [12]) that Probstein obtained by simple superposition virtually identical results to those of Lick. This result is expected in the optically thin and optically thick regimes (page 259 of [12]), but there is no theoretical justification for this in the intermediate regime. In the case of combined conduction an pure scattering, the radiation transfer is uncoupled from the conservation of energy equation. This means that the pure scattering mode cannot affect the conduction mode, since there is no absorption or emission within the medium; and, conversely, it means that the pure scattering radiation transfer is independent of the temperature field (or conduction) within the medium. The purpose of noting these facts is to argue that, if the scattering mode were added to the absorption and conduction modes considered by Lick and Probstein, the good comparison between results (with and without superposition) would still be expected to appertain. Again, this is because the scattering mode is independent of the temperature field which determines the absorption and conduction heat fluxes. A further condition for good applicability of the superposition method is that the surface emittances be high [23]. This is normally the case with insulation test apparatuses.

As we discussed in Section I of this publication, one experimental work on this subject (of Pelanne [4]) indicates that the error due to the superposition assumption is significant (i.e., 5 to 10% of the total thermal conductivity); this is contrary to the above theoretical arguments.

Another experimental work (Pelanne [8]) leads to the opposite conclusion. In this experiment, paper septums are inserted at regular intervals within the sample (which is between hot and cold plates) to block radiation. To understand this, suppose that the total sample width is L_T, and that n septums are placed at intervals of L within the sample. The combined heat transfer between each pair of adjacent septums is the same for each interval, n--due to equilibrium conditions.

$$q = q_1 = k_c \frac{(T_1 - T_c)}{L} + \frac{\sigma(T_1{}^4 - T_c)}{2/\varepsilon - 0.9343 + 3/4\beta L} = q_n = k_c \frac{(T_n - T_{n-1})}{L} + \frac{\sigma(T_n{}^4 - T_{n-1}{}^4)}{2/\varepsilon - 0.9343 + 3/4\beta L}$$

etc., assuming the emissivities of both the septums and the plates are equal to ε. If one adds the q_n's, the "T_n" terms cancel and one can calculate

$$q = k_c \frac{(T_h - T_c)}{nL} + \frac{\sigma(T_h{}^4 - T_c{}^4)}{n(2/\varepsilon - 0.9343 + 3/4\beta L)} .$$

Using the equation, $k = q \dfrac{L}{(T_h - T_c)}$, $k = k_c + \dfrac{4\sigma T^3 L}{2/\varepsilon - 0.9343 + 3/4\beta L}$.

This result for a thick sample with radiation barriers at intervals of L, is the same as the result for a thin sample of thickness L.

The important point is that, if there were a significant coupling term between the radiation and conduction terms for q (i.e., if the assumption of linear superposition leads to significant error), then cancelation of "T_n" terms in the derivation would not be expected to where $\bar{k}_s = \frac{L}{\Sigma(L_i/k_{si})}$.

This equation is recommended for the calculation of the combined conductivity of a thick sample based on measurements of the apparent conductivity of its component layers, in the ASTM C653-70 Test Method [26].

EXPERIMENTAL CONSIDERATIONS

The equations in the Combined Radiation and Conduction Section for the combined thermal conductivity and resistance as a function of sample thickness or density depend on only two parameters to be determined by experiment, namely, the "conduction" conductivity, k_c, and the extinction coefficient, β, $(k_s = \frac{4\sigma\bar{T}^3}{3/4\beta})$. For $\tau o > 1$, the Heaslet-Warming approximation can be used. In this case, the equation for the combined thermal conductivity is

$$k(L) = k_c + \frac{k_s}{1 + (\frac{G}{L})k_s} \tag{33}$$

where $G = \frac{(2/\varepsilon - 0.9343)}{4\sigma\bar{T}^3} = 1.23*$ (See Figure 2a)

The two parameters to be determined by measurement of $(k_t;L)$ are k_c and k_s. (33) is plotted in Fig. 2.

There are several ways to use (33) in practice. The most simple method is suggested as a revision to ASTM C653-70. This method assumes that k_c is predetermined. As was described on page 12, $k_c = a + bD$. In this case, there would be only one unknown (k_s) in (33). Consequently, k_s could be calculated based on a measurment of k_t at a single value of L,

$$k_s = \frac{k - k_c}{1 - \frac{G}{L}(k - k_c)} \tag{34}$$

Note that, if only one measurement were made, one could use (33) to extrapolate k to larger L, assuming that material were homogeneous.

If it were considered preferable to keep k_c as a parameter to be determined by measurement in (33), then there would be several ways to proceed. One method would be to use an iterative numerical technique to fit k_c and k_s to a set of data points, $k_i(L_i)$, in a least-squares manner.

Other methods would utilize a polynomial expansion of (33). If $^{Gk}s/L < 1$, it can be approximated by

$$k(L) = k_c + k_s - (\frac{G}{L})k_s^2 + (\frac{G}{L})^2 k_s^3. \tag{35}$$

One can calculate values of the thickness, above which the various orders of expansion give a better than 1% accuracy. For a density, D, of 0.5 Lb/Ft^3, this thickness is 2.6 In if one keeps the "k_s^2" term and 1.3 In if one keeps the "K_s^3" term.**

Since there are two unknowns in (35), one would need two sets of data points, $k_i(L_i)$, to solve for k_c and k_s. If there were many data points (for many thicknesses or samples of a particular product), one would use Gaussian statistics to calculate the mean and deviation from the mean of the parameters, k_c and k_s.

A third method would be to use the techniques of multiple linear regression to fit data points to a curve.

* For $\varepsilon = 0.9$ and $4\bar{T}^3 = (T_2^2 + T_2^2)(T_1 + T_2)$ with $T_1 = 55°F$; $T_2 = 95°F$; $4\sigma\bar{T}^3 = 1.0461 \frac{Btu}{Hr\text{-}Ft^2\text{-}°F}$

** This is assuming $\beta = 10$ D in units in In^{-1}, which is probably a conservative estimate.

In this case, the equation for k must be linear in the parameters. (33) can be rewriten in the form:

$$k(L) = m - \frac{n}{L} + \frac{q}{L^2} + \ldots\ldots, \tag{36}$$

where $m = k_c + k_s$

$n = Gk_s{}^2$

$q = n(Gk_s) = G^{1/2}n^{3/2}$.

The number of measurements required to determine the parameters is equal to the number of parameters, even though in theory there are only two independent parameters (m and n in this case). It would be easy to fit the parameters to a number of data points, since "canned" programs for linear multiple regression are easily available.

An independent check could be made of the extinction coefficient, using optical experiments [3, 13, 24, 19]. Pelanne [19] pointed out that, if the relationship between optical parameters (e.g., the extinction coefficient and the transmission) is well understood, then rapid optical checks could be used in quality control of insulation production, instead of time-consuming heat-flow-meter checks.

SUMMARY AND RECOMMENDATIONS FOR FUTURE WORK

1. A new approximate expression is suggested for the radiative heat flux as a function of thickness. This approximation is compared with other approximations.

2. Assuming a linear superposition of the radiative and conductive modes of heat transfer, various approximations (along with the corresponding curves and regimes) are presented for the following thermal quantities: the combined thermal resistance as a function of thickness and the combined thermal conductivity as a function of thickness and of density. These equations are suggested as a means to predict the "thickness effect".

3. Recommendations are made for improvements of the ASTM Test Methods C653-70 (pages 7, 9 and 10) and C518-76$^{\varepsilon}$ or C177-76$^{\varepsilon}$ (pages 6 and 7). The significance of these recommendations lies in the fact that the "thickness effect" is included in the expressions for the combined thermal conductivity and resistance. The hope then is that, when the parameters describing the thickness effect are better understood, it should be possible to predict with a commensurate accuracy the combined thermal resistance at large thicknesses based on measurements at smaller thicknesses. This would, of course, result in more economical quality control and product certification.

4. A discussion is given concerning the plausibility of the assumption of linear superposition of conductive and radiative modes, based on currently available literature.

5. An expression is obtained for the combined thermal resistance of multi-layer insulation in the case of combined conduction and radiation.

6. A discussion is given of the utility to the researcher of the equations for the combined thermal resistance and conductivity. In addition, physical orders or magnitude of the terms in the equations are given whenever possible, again for the benefit of the researcher.

7. Recommendations are made to investigate the isotropic and anisotropic radiation scattering and the radiation absorption, as a function of wave length, of fiber size and composition and of bulk density. The hope is that a better understanding of the optical parameters will make possible the use of rapid optical measurements in the quality control of low-density insulation production. For example, it might be possible to determine the optical parameter (β) with a rapid measurement. Then this parameter along with a measurement at a small thickness (e.g., 3") of the apparent thermal resistance might make it possible to predict the apparent thermal resistance at a large thickness (e.g., 12"), with a 2% accuracy. In addition, there should be further work on the so-called "k-interaction" term.

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TABLE 1

Comparison of "Exact" Dimensionless Heat Flux with Approximations

τ_0	Q_{exact}	Q_{TH}	(Δ%)	Q_{HW}	(Δ%)	Q_{EK}	(Δ%)
.1	.9157	.9191	(+ .4%)	.8766	(-4.4%)	.9302	(+1.6%)
.2	.8491	.8511	(+ .2%)	.8226	(-3.2%)	.8696	(+2.4%)
.3	.7934	.7934	(0%)	.7748	(-2.4%)	.8163	(+2.9%)
.4	.7458	.7443	(- .2%)	.7322	(-1.9%)	.7692	(+3.1%)
.5	.7040	.7017	(- .3%)	.6941	(-1.4%)	.7273	(+3.3%)
.6	.6672	.6645	(- .4%)	.6598	(-1.1%)	.6899	(+3.4%)
.8	.6046	.6023	(- .4%)	.6003	(- .7%)	.6250	(+3.4%)
1.0	.5532	.5515	(-0.3%)	.5508	(- .4%)	.5714	(+3.3%)
1.5	.4572	.4565	(-0.15%)	.4565	(- .15%)	.4705	(+2.9%)
2.0	.3900	.3898	(-0.15%)	.3897	(-0.08%)	.4000	(+2.6%)
2.5	.3401	← Same		← Same		.3478	(+2.3%)
3.0	.3016	↓		↓		.3077	(+2.0%)
16.5	.0744					.0748	(+ .5%)

TABLE 2

Values of Representative Thickness, Lrp*

Density (Lb/ft^3)	β' = 20 Lrp	β' = 20 [k_s]**	β' = 15 Lrp	β' = 15 [k_s]	β' = 10 Lrp	β' = 10 [k_s]
0.3	4.8"	[.23]	11.7"	[.31]	20.0"	[.46]
0.5	3.5"	[.14]	5.6"	[.19]	10.1"	[.28]
0.75	1.8"	[.09]	3.0"	[.12]	5.6"	[.19]
1.0	1.1"	[.03]	1.8"	[.09]	3.5"	[.14]
2.0	0.3"	[.03]	0.5"	[.05]	1.1"	[.07]

* $$Lrp = G\, k_s \left(\frac{1}{.02} \left(\frac{k_s}{k_c + k_s} \right) - 1 \right)$$

** $$k_s = \frac{4\sigma \bar{T}^3}{3/4\beta' D} \quad (\text{units of } \frac{\text{Btu-In}}{\text{Hr-ft}^2\text{-}°\text{F}})$$

k_c = 0.18 (same units); G = 1.23 (in units of $\frac{\text{Hr-Ft}^2\text{-}°\text{F}}{\text{Btu}}$).

Fig. 1a

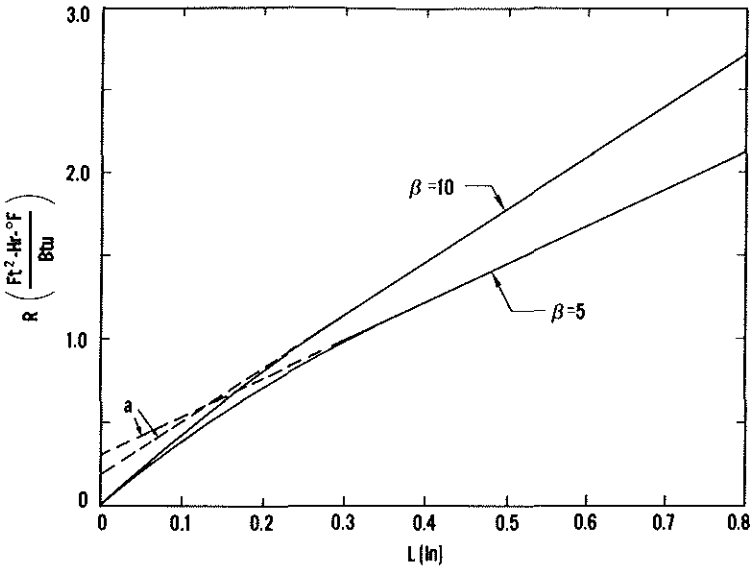


Fig. 1b

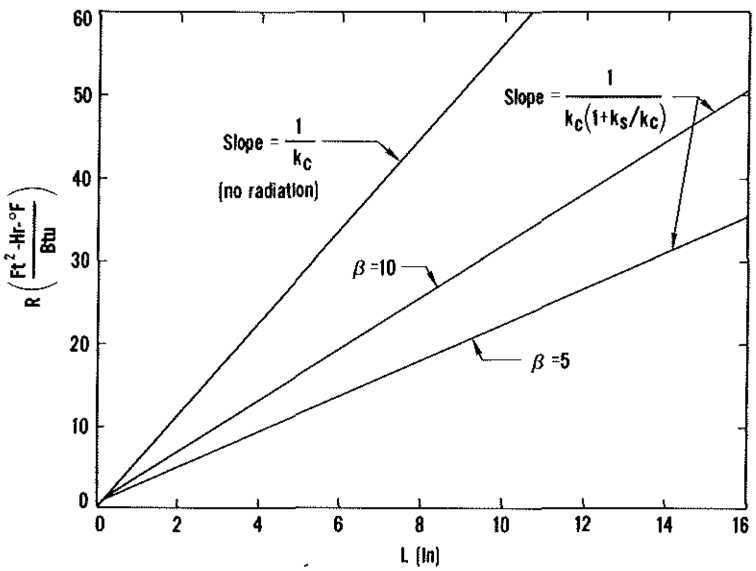


Fig. 2a

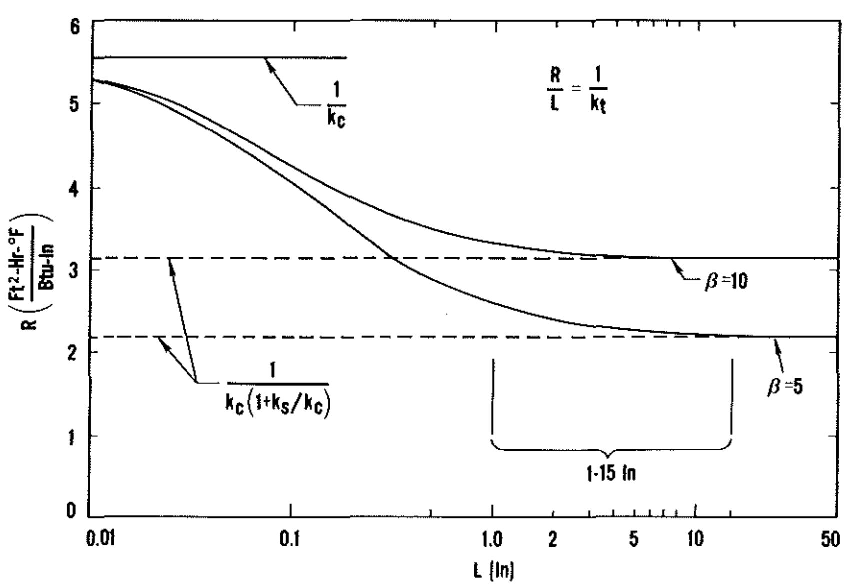


Fig. 2b

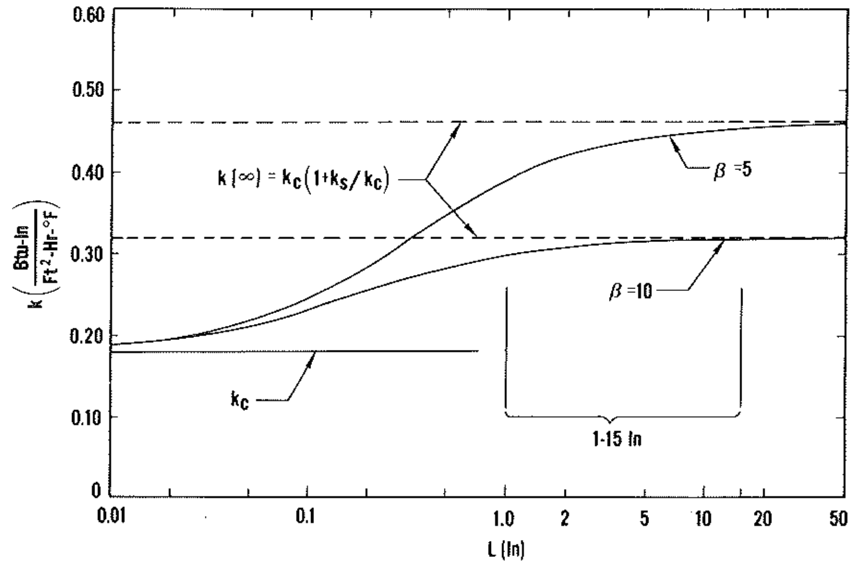
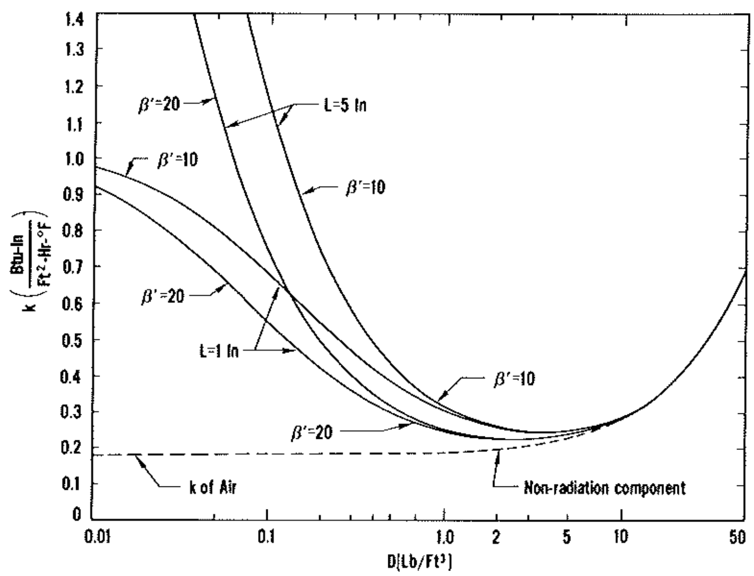


Fig. 3



Multi-slab Configuration

Fig. 4